

RESPONSE TO CHANWOO LEE'S 'HOW WE LEARNED TO STOP WORRYING AND LOVE 'MODEL' IN MATHEMATICS'

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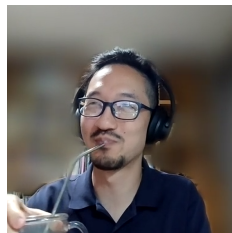


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- Thanks to the organisers and Chanwoo for the great talk!
- It's been busy here at the start of the Semester, I'm sorry to not have come to more of the conference



- As you've seen, Chanwoo Lee's 'How We Learned to Stop Worrying and Love 'Model' in Mathematics' has the following rough **structure**:
 1. Present the notion of a **relative consistency proof**.
 2. Outline Frege's **complaint** about Hilbert's relative consistency proof (the **Change of Subject Objection**).
 3. Point out that mathematicians are, largely speaking, **unmoved** by the response.
 4. Argue that two **natural** responses; **set-theoretic reductionism** and **structuralism** don't have satisfactory responses.
 5. Argue instead that by treating models as **analogous** with use in **other** sciences provides a better explanation.

CHANGE OF SUBJECT OBJECTION (CoS)

The Change of Subject (CoS) objection: Hilbert's RC proof does **not succeed** because the proof concerns a **subject matter** which is **distinct** from that of the target theory.

- I do **not** want to challenge Chanwoo's characterisation of Frege's complaint.
- Frege scholars are **better** placed to do so than I, and in any case in the paper he acknowledges **other** interpretations.
- I think the paper makes a **great** case for its main points.
- My main question: **In what sense do these points depend on the “everyday” sense of model?**

- Penelope Maddy (in [Maddy, 2017] and [Maddy, 2019]) discusses **purposes** of **mathematical foundations**.
- Examples:
- **Metamathematical Corral**. Provide a theory in which metamathematical investigations of **relative provability** and **consistency strengths** can be easily conducted.
- **Generous Arena**. Find *representatives* for our usual mathematical structures (e.g. \mathbb{N} , \mathbb{R}) using our theory of sets.
- **Risk Assessment**. Provide a degree of confidence in theories **commensurate** with their consistency strength.
- **Essential Guidance**. Capture the fundamental character of mathematics as it's **actually done**, ... guide mathematicians toward the **truly important concepts** and **structures**, without getting bogged down in **irrelevant details**.

QUESTION.

How does the present proposal **relate** to these criteria?

- Here's what I think explains the **indifference** to the CoS-objection.
- In many cases, you just care whether your theory is **consistent** (**Risk Assessment** given by **Metamathematical Corral** and **Generous Arena**).
- An RCP says you can (up to a point) just **go ahead** and use your language **without worrying** if it's consistent (cf. Mac Lane and Grothendieck using universes).
- **Any** old model will do!

QUESTION.

The above didn't seem to really depend on an analogy to the idea of **model** in the everyday scientific sense — it seems like a purely **mathematical** explanation. What's the response?

- There's an **exception**, when we actually want **Essential Guidance**.
- But here mathematicians **do** care!
- Look at the **revolt** against old-school set-theoretic foundations (in favour of the use, for example, of category-theoretic tools).
- And here it **does** seem that there's a closer analogy to the ordinary sciences — we care **how good** a model is.
- This looks like the modified-Frege objection... but why is it depending on the “**everyday scientific sense**” of modelling? Isn't this just a **mathematical** point?

QUESTION.

Why are these issues where we take notions from **ordinary science** rather than general **good modelling procedures** (whether scientific or no)? What is the purported position that we're arguing **against**?

(Almost no-one likes set-theoretic reductionism, and structuralism seems like a bolt-on.)

- I have **many** further questions — I **really** enjoyed reading the paper.
- **Sadly** I've got to leave it there...
- But I think that the paper asks **all the right** questions, in particular...

QUESTION.

What do we want from **good** models anyway? Is there a **difference** between the (everyday) **sciences** and **mathematics**?

- **Bonus questions** in the **unlikely** event there's time.
- **Question.** What do different **kinds** of interpretability tell us? (e.g. bi-interpretability gets us some kind of **equivalence**?)
- **Question.** What about the use of **unintended** models? (e.g. overspill arguments, use of forcing arguments in set theory)
- **Question.** How to think of **partial** modelling? Normally we think we have **full** satisfaction of the axioms. But maybe related to unintended features of models. Can also consider e.g. models of the Σ_n -fragment of ZFC.
- **Question.** What is the **ontology** of models (sui generis entities, [Antos, F])?
- **Question.** Don't we often consider modelling structures out of the **same** entities (e.g. model theory of sets)?

- **Question.** Don't models in mathematics serve to highlight what is **expressible in a logic** in a way that is not really shared by the physical sciences?
- **Question.** Doesn't model theory in mathematics also concern notions of **niceness of theory** (e.g. Morley Categoricity, dimension, stability theory,..) rather than just RCPs?

Thanks for listening!

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