Response to Chanwoo Lee's 'How We Learned to Stop Worrying and Love 'Model' in Mathematics'

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- Thanks to the organisers and Chanwoo for the great talk!
- It's been busy here at the start of the Semester, I'm sorry to not have come to more of the conference

- As you've seen, Chanwoo Lee's 'How We Learned to Stop Worrying and Love 'Model' in Mathematics' has the following rough structure:
    1. Present the notion of a relative consistency proof.
    2. Outline Frege's complaint about Hilbert's relative consistency proof (the **Change of Subject Objection**).
    3. Point out that mathematicians are, largely speaking, unmoved by the response.
    4. Argue that two natural responses; **set-theoretic reductionism** and **structuralism** don't have satisfactory responses.
    5. Argue instead that by treating models as analogous with use in other sciences provides a better explanation.

Change of Subject Objection (CoS)

The Change of Subject (CoS) objection: Hilbert's RC proof does not succeed because the proof concerns a subject matter which is distinct from that of the target theory.

- I do not want to challenge Chanwoo's characterisation of Frege's complaint.
- Frege scholars are better placed to do so than I, and in any case in the paper he acknowledges other interpretations.
- I think the paper makes a great case for its main points.
- My main question: **In what sense do these points depend on the "everyday" sense of model**?

- Penelope Maddy (in [Maddy, 2017] and [Maddy, 2019]) discusses purposes of mathematical foundations.
- Examples:
- **Metamathematical Corral.** Provide a theory in which metamathematical investigations of relative provability and consistency strengths can be easily conducted.
- **Generous Arena.** Find *representatives* for our usual mathematical structures (e.g. $\mathbb{N}$, $\mathbb{R}$) using our theory of sets.
- **Risk Assessment.** Provide a degree of confidence in theories commensurate with their consistency strength.
- **Essential Guidance.** Capture the fundamental character of mathematics as it's actually done, ... guide mathematicians toward the truly important concepts and structures, without getting bogged down in irrelevant details.

Question.

How does the present proposal relate to these criteria?

- Here's what I think explains the indifference to the CoS-objection.
- In many cases, you just care whether your theory is consistent (**Risk Assessment** given by **Metamathemtical Corral** and **Generous Arena**).
- An RCP says you can (up to a point) just go ahead and use your language without worrying if it's consistent (cf. Mac Lane and Grothendieck using universes).
- Any old model will do!

Question.

The above didn't seem to really depend on an analogy to the idea of model in the everyday scientific sense — it seems like a purely mathematical explanation. What's the response?

- There's an exception, when we actually want **Essential Guidance**.
- But here mathematicians do care!
- Look at the revolt against old-school set-theoretic foundations (in favour of the use, for example, of category-theoretic tools).
- And here it does seem that there's a closer analogy to the ordinary sciences — we care how good a model is.
- This looks like the modified-Frege objection... but why is it depending on the "everyday scientific sense" of modelling? Isn't this just a mathematical point?

Question.

Why are these issues where we take notions from ordinary science rather than general good modelling procedures (whether scientific or no)? What is the purported position that we're arguing against?
(Almost no-one likes set-theoretic reductionism, and structuralism seems like a bolt-on.)

- I have many further questions — I really enjoyed reading the paper.
- Sadly I've got to leave it there...
- But I think that the paper asks all the right questions, in particular...

Question.

What do we want from good models anyway? Is there a difference between the (everyday) sciences and mathematics?

- Bonus questions in the unlikely event there's time.
- **Question.** What do different kinds of interpretability tell us? (e.g. bi-interpretability gets us some kind of equivalence?)
- **Question.** What about the use of unintended models? (e.g. overspill arguments, use of forcing arguments in set theory)
- **Question.** How to think of partial modelling? Normally we think we have full satisfaction of the axioms. But maybe related to unintended features of models. Can also consider e.g. models of the Σ_n-fragment of ZFC.
- **Question.** What is the ontology of models (sui generis entities, [Antos, F])?
- **Question.** Don't we often consider modelling structures out of the same entities (e.g. model theory of sets)?

- **Question.** Don't models in mathematics serve to highlight what is expressible in a logic in a way that is not really shared by the physical sciences?
- **Question.** Doesn't model theory in mathematics also concern notions of niceness of theory (e.g. Morley Categoricity, dimension, stability theory,..) rather than just RCPs?

Thanks for listening!

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