

GÖDELIAN PLATONISM RE-IMAGINED

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- **Note about the talk:** This is all very **preliminary**, and I’m **very** happy to be set straight!

MAIN AIMS.

1. Argue that there's an **idea** coming from Gödel's work (and in particular the notebooks) that suggests a kind of **representationalism** about mathematical epistemology.^a
2. Argue that this is an **attractive** view (at least for certain applications).

^a**Note:** This terminology should probably be changed, as there's a kind of **representationalism** in the philosophy of mind/cognitive science that I'm (probably) not intending here.

INTRODUCTION

A TENSION IN GÖDEL'S PLATONISM

GÖDELIAN REPRESENTATIONALISM

THE NOTEBOOKS

APPLICATIONS

CONCLUSIONS AND OPEN QUESTIONS

- There's a standard **bad** view of Gödel that pervades a lot of the (older) literature.

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- It arises from the following quotation in the 1964 version of "What is Cantor's continuum problem?"

*But, despite their remoteness from sense experience, we do have something like a **perception** of the objects of set theory, as is seen from the fact that the axioms **force themselves upon us as being true**. I don't see any reason why we should have **less confidence** in this kind of perception, i.e., in mathematical intuition, than in sense perception.*

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- “a logician par excellence but a **philosophical fool**” (James 1992, p. 131)

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*“The result of our previous discussion is that our axioms, if interpreted as meaningful statements, necessarily presuppose a **kind of Platonism**, which **cannot satisfy any critical mind** and which does not even produce the conviction that they are consistent.”*

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- Here’s the Gibbs lecture anyway (1951):

*...the Platonistic view is the only one tenable. Thereby I mean the view that mathematics describes a non-sensual reality, which exists independently both of the acts and the dispositions of the human mind and is only perceived, and **probably perceived very incompletely**, by the human mind*

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- **Gödel's Tension.** On the one hand he says that we have a strong enough faculty of "perception" to get axioms that **force themselves on us as true**, but on the other hand he thinks that such perception may be **incomplete** and the account **cannot satisfy any critical mind**.

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- We might even think that perception is just a way of forming very good representations.

Even in the 1964 paper Gödel writes:

Mathematical intuition need not be conceived of as a faculty giving an immediate knowledge of the objects concerned. Rather it seems that, as in the case of physical experience, we form our ideas also of those objects on the basis of something else which is immediately given. Only this something else here is not, or not primarily, the sensations.

That something besides the sensations actually is immediately given follows (independently of mathematics) from the fact that even our ideas referring to physical objects contain constituents qualitatively different from sensations or mere combinations of sensations, e.g., the idea of object itself, whereas, on the other hand, by our thinking we cannot create any qualitatively new elements but only reproduce and combine those that are given...

*It by no means follows, however, that the data of this second kind, because they cannot be associated with actions of certain things upon our sense organs, are something purely subjective, as Kant asserted. Rather they, too, may **represent** an aspect of objective reality, but, as opposed to the sensations, their presence in us may be due to **another kind of relationship** between ourselves and reality. (p. 268 of the 1967 paper)*

*Here once again there is no need to object to what Godel says in advance of an account of what this ‘other kind of relationship’ is. Unfortunately that is just what he does not supply: the paragraph ends where my quotation does, and **the thought is not developed any further**. (p. 341, Potter “Was Gödel a Gödelian Platonist”, 2001)*

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- But **how** does this representation work?

Returning again to the Gibbs lecture:

*The truth, I believe, is that these concepts form an objective reality of their own, which we cannot create or change, but only perceive **and describe***

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- We have epistemic access to mathematical objects by **describing** them using theories.
- Perhaps we have something analogous to **description**.
- There are **structural** and **representational similarities** between things we have direct access to, and things we don't.

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- Perhaps we can gain knowledge of platonic reality by providing suitable **visual pictures**.
- These need not be **perfect** (e.g. a triangle).
- But they can do a job of helping us to **latch on** to particular ideas.

- **Example 2:** Representing **complexity of a concept** via **complexity of a formula**.
- The formula complexity “encodes” the amount of “**completed searches**” necessary.
- There is a correspondence between the **syntax used** and the **difficulty** of the definition we’re interested in.

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- More suggestions **very welcome!**

Remark (Psychology): According to Aristotle, [209] fantasy is an ability to judge at a low level. (The only ability to judge that animals have, insofar as they can only imagine what they expect?)

A side note:

The same is the case for humans with respect to mathematical propositions (but here as well, it is possible to imagine the opposite symbolically).

Remark (Psychology): In order to learn, it is very important to choose the “right” symbols and to hold on to them (for generating sensuous associations) and for one not to choose too many symbols. The opposite holds for mathematical and positivist work: wrong, changing and a plethora of symbols are chosen. A single correct symbol means tremendously much. (p. 203 Band IV)

*Knowledge is a **picture of reality**, since it is only by “pictures” that we can prepare ourselves. Turn the encounter with the **unknown** into an encounter with the **known**. Overcome unfamiliarity or moderate it (p. 161, Band I)*

*Knowledge is a kind of picture. The purpose of **representation** in everyday life is to replace reality. (p. 168, Band I)*

*To explain something means to express something that we usually express using certain words with the help of other words in such a way that the description becomes perfect (for example: Replacing warmth by kinetic energy). The main point of knowledge by description as well as knowledge via explanation is to **describe something new with something known**. The new is reduced to the known. (p. 164, Band I)*

*There are only **very few things that we can perceive** (e.g. numbers only up to about 10), namely: 1.) due to too much complexity, 2.) due to too great a distance” (e.g. other people’s emotions or the simple, but very high ideas). This means that no possible intensional objects correspond to these things.*

*But for other things, we have a “substitute” in us (namely other intensional objects which we **can perceive** and which can **represent them**), for example in the case of too much complexity, the object is replaced by a simple “sign”. (That is the function of symbols.)
...a deduction allows one to perceive a reality **indirectly***

By handing over the “substitute” for non-perceivable objects (the symbol or concept combination) to memory, together with the elementary states of affairs that are noticed about it, I have in a certain way “created” in me the object that was not in me before — made a substitute for it directly perceivable. And this substitute is useful (or perfect) insofar the relations of the object are also reflected in the substitute in such a way that they are directly perceivable (since the corresponding substitute relations are directly perceivable). However, this “creation” of the non-perceivable objects in oneself is not arbitrary, but rather uniquely determined, similar to the way facial expressions are a “sign” for the anger of another person. (Band I, 189–190)

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- But I'm not concerned with **meaning** here, but rather **aboutness** and **knowledge**.
- Do these all come down to the same thing? I'm not sure...

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- But it's not some kind of **mystical** connection.

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- It's not clear that the more "transcendental" sense is **that** interesting, unless it is **also** tractable.
- This is where a lot of the action is in the philosophy of set theory, and perhaps where some **progress** might get made.

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- And I think it's worth thinking about them as a way of **naturalising** mathematical epistemology.
- There is a **meta**-worry here, but let's see if that comes up in discussion...

Thanks for listening!