Gödelian platonism re-imagined

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- First, an apology to everyone and (in particular Yuxuan Liu).
- I was supposed to give this talk at the Gödel workshop but caught a bug on the ride over to Singapore.
- I know it's annoying to have people cancel at the last minute, and it looked like a great conference.

- I was saved a bit of embarrassment though.
- The title of this paper was originally "Was Gödel a gödelian platonist" a play on Elaine Landry's recent booklet "Plato Was *Not* a Mathematical Platonist"...
- ...with a view to eventually arguing that Gödel was not a gödelian platonist.
- Tim Button pointed out to me that Michael Potter had written a paper arguing exactly this, with almost exactly the same title.
- So a change of title was necessary!
- **Note about the talk:** This is all very preliminary, and I'm very happy to be set straight!

Main Aims.

1. Argue that there's an idea coming from Gödel's work (and in particular the notebooks) that suggests a kind of representationalism about mathematical epistemology.[a]
2. Argue that this is an attractive view (at least for certain applications).

[a]**Note:** This terminology should probably be changed, as there's a kind of representationalism in the philosophy of mind/cognitive science that I'm (probably) not intending here.

Introduction
A tension in Gödel's platonism
Gödelian Representationalism
The notebooks
Applications
Conclusions and open questions

- There's a standard bad view of Gödel that pervades a lot of the (older) literature.
- It arises from the following quotation in the 1964 version of "What is Cantor's continuum problem?"

But, despite their remoteness from sense experience, we do have something like a perception of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e., in mathematical intuition, than in sense perception.

- Gödel was definitely a platonist, in the sense that he thought that the objects of mathematics exist (and aren't mental constructions etc.).
- But he is often accused of thinking that we have some quasi-perceptual faculty that grants us epistemic access to these objects.
- For this reason, Gödel is often accused of proposing a mystical (and non-naturalisable) account of mathematical epistemology.
- Some point to this as being theologically analogous (e.g. Chihara, 1990).
- Others are harsher still.
- "a logician par excellence but a philosophical fool" (James 1992, p. 131)

But in his 1933 *The present situation in the foundations of mathematics* he strikes a very different tone:

> *"The result of our previous discussion is that our axioms, if interpreted as meaningful statements, necessarily presuppose a kind of Platonism, which cannot satisfy any critical mind and which does not even produce the conviction that they are consistent."*

- This seems very different.
- Assuming that the platonism being alluded to is one including the "mystical eye"account of mathematical epistemology, he seems to be outright rejecting it here.
- Maybe you want to argue that those appear very far apart. Perhaps he just changed his mind — no mystery there.
- I don't find it plausible that he just straightforwardly changed his mind, he seems to repeatedly revisit these issues (looking at the notebooks too, as we'll see).
- Here's the Gibbs lecture anyway (1951):

...the Platonistic view is the only one tenable. Thereby I mean the view that mathematics describes a non-sensual reality, which exists independently both of the acts and the dispositions of the human mind and is only perceived, and probably perceived very incompletely, by the human mind

- Here he allows for some sense perception, but argues that it is very incomplete.
- This seems a far cry from the axioms forcing themselves on us as true.
- I thus think that there's a tension in Gödel's thought.
- **Gödel's Tension.** On the one hand he says that we have a strong enough faculty of "perception" to get axioms that force themselves on us as true, but on the other hand he thinks that such perception may be incomplete and the account cannot satisfy any critical mind.

- Here's a general view concerning how we're able to have thoughts about things in the world.
- Suppose I am thinking about Toffee the cat.
- One way is to see Toffee.
- But I could also think about Toffee without directly seeing her.
- I form some kind of representation of her in my mind.
- Presuming that my representation is good enough, this can be about her, no problem.
- We might even think that perception is just a way of forming very good representations.

Even in the 1964 paper Gödel writes:

> *Mathematical intuition need not be conceived of as a faculty giving an immediate knowledge of the objects concerned. Rather it seems that, as in the case of physical experience, we form our ideas also of those objects on the basis of something else which is immediately given. Only this something else here is not, or not primarily, the sensations.*

That something besides the sensations actually is immediately given follows (independently of mathematics) from the fact that even our ideas referring to physical objects contain constituents qualitatively different from sensations or mere combinations of sensations, e.g., the idea of object itself, whereas, on the other hand, by our thinking we cannot create any qualitatively new elements but only reproduce and combine those that are given...

It by no means follows, however, that the data of this second kind, because they cannot be associated with actions of certain things upon our sense organs, are something purely subjective, as Kant asserted. Rather they, too, may represent an aspect of objective reality, but, as opposed to the sensations, their presence in us may be due to another kind of relationship between ourselves and reality. (p. 268 of the 1967 paper)

Here once again there is no need to object to what Godel says in advance of an account of what this 'other kind of relationship' is. Unfortunately that is just what he does not supply: the paragraph ends where my quotation does, and the thought is not developed any further. (p. 341, Potter "Was Gödel a Gödelian Platonist", 2001)

- I think Gödel may have considered something similar to representationalism.
- (Or, at least, I'm happy to just explore the view. Both questions seem interesting.)
- We don't get direct perception of mathematical objects.
- But what we can do is form representations of mathematical objects, one's that can conform better or worse to the way the world is.
- But how does this representation work?

Returning again to the Gibbs lecture:

> *The truth, I believe, is that these concepts form an objective reality of their own, which we cannot create or change, but only perceive and describe*

- An analogy from the literature on structuralism.
- We have epistemic access to mathematical objects by describing them using theories.
- Perhaps we have something analogous to description.
- There are structural and representational similarities between things we have direct access to, and things we don't.

- **Example 1:** Visual thinking in mathematics.
- Perhaps we can gain knowlege of platonic reality by providing suitable visual pictures.
- These need not be perfect (e.g. a triangle).
- But they can do a job of helping us to latch on to particular ideas.

- **Example 2:** Representing complexity of a concept via complexity of a formula.
- The formula complexity "encodes" the amount of "completed searches" necessary.
- There is a correspondence between the syntax used and the difficulty of the definition we're interested in.

- Can we find evidence for this in Gödel?
- I think it might be interesting here to revisit some remarks from the notebooks.
- More suggestions very welcome!

Remark (Psychology): According to Aristotle, [209] fantasy is an ability to judge at a low level. (The only ability to judge that animals have, insofar as they can only imagine what they expect?
A side note:
The same is the case for humans with respect to mathematical propositions (but here as well, it is possible to imagine the opposite symbolically).

Remark (Psychology): In order to learn, it is very important to choose the "right" symbols and to hold on to them (for generating sensuous associations) and for one not to choose too many symbols. The opposite holds for mathematical and positivist work: wrong, changing and a plethora of symbols are chosen. A single correct symbol means tremendously much. (p. 203 Band IV)

Knowledge is a picture of reality, since it is only by "pictures" that we can prepare ourselves. Turn the encounter with the unknown into an encounter with the known. Overcome unfamiliarity or moderate it (p. 161, Band I)

Knowledge is a kind of picture. The purpose of representation in everyday life is to replace reality. (p. 168, Band I)

To explain something means to express something that we usually express using certain words with the help of other words in such a way that the description becomes perfect (for example: Replacing warmth by kinetic energy). The main point of knowledge by description as well as knowledge via explanation is to describe something new with something known. The new is reduced to the known. (p. 164, Band I)

There are only very few things that we can perceive (e.g. numbers only up to about 10), namely: 1.) due to too much complexity, 2.) due to too great a distance" (e.g. other people's emotions or the simple, but very high ideas). This means that no possible intensional objects correspond to these things.

But for other things, we have a "substitute" in us (namely other intensional objects which we can perceive and which can represent them), for example in the case of too much complexity, the object is replaced by a simple "sign". (That is the function of symbols.)
...a deduction allows one to perceive a reality indirectly

By handing over the "substitute" for non-perceivable objects (the symbol or concept combination) to memory, together with the elementary states of affairs that are noticed about it, I have in a certain way "created" in me the object that was not in me before — made a substitute for it directly perceivable. And this substitute is useful (or perfect) insofar the relations of the object are also reflected in the substitute in such a way that they are directly perceivable (since the corresponding substitute relations are directly perceivable). However, this "creation" of the non-perceivable objects in oneself is not arbitrary, but rather uniquely determined, similar to the way facial expressions are a "sign" for the anger of another person.(Band I, 189–190)

- Gödel makes it very clear at certain points in Band IV, that the meaning relation is not a representation relation.
- But I'm not concerned with meaning here, but rather aboutness and knowledge.
- Do these all come down to the same thing? I'm not sure...

- I want to provide two applications.
- The first is just to dissolve Gödel's tension.
- We do indeed have something like sense perception.
- The ability to form representations that may conform better or worse to the way mathematical reality is.
- This is something that could be naturalised (though that looks like a very hard problem).
- But it's not some kind of mystical connection.

- Second application is to the philosophy of set theory (very non-Gödelian!).
- Perhaps there is one sense, in which a sentence like CH does have a definite truth value.
- e.g. If I make a quantified statement about the sets with no specific representation in mind.
- But perhaps there's another, conformity with my representation, where it doesn't (or we can have some relativism here).
- This is a bit like a kind of enriched if-then-ism.
- It's not clear that the more "transcendental" sense is that interesting, unless it is also tractable.
- This is where a lot of the action is in the philosophy of set theory, and perhaps where some progress might get made.

- All in all, I'm not really sure if Gödel was a representationalist in the manner I've suggested.
- But maybe he did play around with these ideas.
- And I think it's worth thinking about them as a way of naturalising mathematical epistemology.
- There is a meta-worry here, but let's see if that comes up in discussion...

Thanks for listening!