Mathematics and the Aim of Inquiry

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- Currently, we want to find out if every even number is the sum of two primes.

Question.

When we inquire into some subject matter or other, what are we trying to do? And what are the norms governing inquiry?

- Thus far, mathematics has been largely ignored in the inquiry literature (let me know if I'm wrong!).
- This is unusual since mathematics is very logically hygenic.
- We can literally prove proof-theoretic facts (e.g. that certain things are unprovable)!

Main claims.

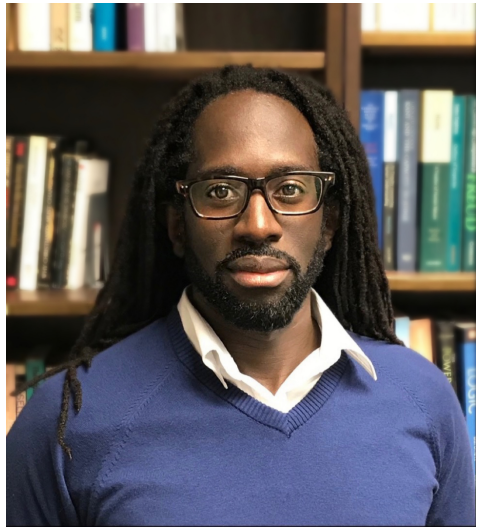
1. Studying inquiry in mathematics supports the **Epistemic Improvement Account** of inquiry.
2. It also (relatedly) suggests some commonly suggested norms surrounding inquiry are incorrect.

- First some set-up.
- As I'll talk about things, there are three important classes for inquiry:
  1. Propositions.
  2. Questions.
  3. Subject matters.

- So we have the proposition that *it is sunny outside* (P), the question of whether P (or what the weather is like), and the subject matter (the weather outside right now).
- We can have propositional attitudes (e.g. belief) towards propositions and interrogative attitudes (e.g. regarding as open, inquiring into) questions.
- Primarily the literature has focused on inquiry into questions, but some also consider inquiry into subject matters e.g. [Kelp, 2021].
- I'll follow this, but the division between inquiring into subject-matters and questions will pop up later.

- A central question in the inquiry literature concerns the aim of inquiry.
- What exactly are we trying to do when we inquire?
- **Knowledge account.** One's inquiry into whether P aims at coming to know one of P or $\neg P$. [Kelp, 2021, p. 11]
- There's also the justified true belief account and the true belief accounts, but let's put these to one side.

Epistemic improvement account. ...one is inquiring into some question, Q, only if one is gathering or analysing information that, from one's perspective, may potentially bear on answering Q with the aim of improving one's epistemic standing with respect to Q [Archer, 2021, pp. 96–97]



Quietist account. ...the quietism I am calling for is about 'the aim of inquiry'. To adopt this sort of quietism is not to deny that inquirers are typically trying to get new knowledge or improve their epistemic standings. But the 'typically' matters... Part of my quietistic thought is that we don't need to say. Instead I think we can theorize directly about the structure of inquiry, the norms of inquiry, and the goals of individual inquirers... [Friedman, 2023, pp. 2–3]

- Let's pause to refine what I'm about to argue.
- The aim of inquiry is naturally linked to the norms you might pick (we'll see some of these in a second).
- I'm going to argue that **if** you think inquiry has an aim, **then** you should hold the **epistemic improvement account**.
- But even if you're a quietist (a position I have some sympathy with), some of the norms commonly suggested are incorrect.
- Let's examine some of these norms now.

- The first is what I'll call the **Soundness Norm**.
- **Observation.** Some questions we can see are bad or faulty.
- **Question (Jefferson's Ferrari).** What colour was Thomas Jefferson's Ferrari? [Friedman, 2017, p. 315]
- **Soundness.** Let us say that A regards Q as sound iff A thinks that Q has a correct answer.

Soundness Norm.

In order to have an interrogative attitude towards Q at t, I should regard Q as sound. [Friedman, 2017, pp. 315–316]

- The next is the **Ignorance Norm**.
- Can I inquire into whether $2 + 2 = 4$?
- Natural thought: **No!** You already know that $2 + 2 = 4$.

Ignorance Norm

If A knows the answer to Q, then it is epistemically inappropriate for A to hold an interrogative attitude towards Q. [Friedman, 2017, p. 310]

Apparent Ignorance Norm

If A is aware that they know the answer to Q, then it is epistemically inappropriate for A to hold an interrogative attitude towards Q. [Friedman, 2017, p. 312–313]

B-ignorance Norm

If A believes that they know the answer to Q, then it is epistemically inappropriate for A to hold an interrogative attitude towards Q.

- I think all these norms are incorrect, and I think mathematics can show us how/why.
- Part of this has to do with the fact that I think that the **Epistemic Improvement Account**, if any, is correct, which in turn suggests the incorrectness of the relevant norms.
- Towards the end I will tentatively suggest a norm of my own.

- My argumentative strategy will be time-honoured method: "If all you have is a hammer everything looks like a nail."
- I'll talk about set theory a bunch.
- But I think the problem is quite general...

- The first point: Consideration of unsound questions is all over the place in mathematics.
- **Question.** Is the cofinality of the least Berkeley cardinal ω?
- I believe (indeed I take myself to know) that this question is unsound.
- This is because since the Axiom of Choice is true, there are no Berkeley cardinals.
- This is going to be very general: We are often considering the properties of objects we take to be probably non-existent.
- By considering such propositions, we are inquiring into questions we take to be unsound.

- **The natural response:** This is sophistry! When I consider the cofinality of the least Berkeley cardinal, I'm considering whether it can have the property in some model of ¬AC!
- **First rejoinder:** This isn't so clear to me. For sure that's a question I can inquire into, but I can also inquire into the cofinality of the least Berkeley cardinal.
- Perhaps I take myself to know AC, but defeasibly so.
- **Second rejoinder:** If I'm allowed to have this reinterpretation of this question, why not for Jefferson's Ferrari? (I mean what did Jefferson have in a close possible world...)
- Perhaps, all things considered, Jefferson's Ferrari is lime green.

- Next: Indeterminacy.
- Let's suppose you believe that statements independent of ZFC are indeterminate.
- You can still inquire into some statement you believe to be indeterminate.
- Suppose I am hunting for a proof that some ϕ is indeterminate, and I believe it to be so.
- On the assumption that I believe that questions with indeterminate answers are not sound (more on this in a second), and that searching for an independence proof is inquiring into whether ϕ, then we have a straightforward violation of the **Soundness Norm**.

- Perhaps instead we think think that we need a better analysis of soundness
- For example: [Ferrari and Incurvati, 2021] suggest a distinction between polar (i.e. bivalent) questions, and sound questions.
- The answer to CH might be non-polar but nonetheless sound.
- They use the example of [Hamkins, 2012] to illustrate this.

But Hamkins' version of indeterminacy is not the only one on the market, Shelah has a more "measure-theoretic" approach:

> *....I do not agree with the pure Platonic view that the interesting problems in set theory can be decided, we just have to discover the additional axiom. My mental picture is that we have many possible set theories, all conforming to ZFC. I do not feel "a universe of ZFC" is like "the sun", it is rather like "a human being" or "a human being of some fixed nationality"...*
> *...You may think "does CH hold?" being like "can a typical American be Catholic". [Shelah, 2002, p. 12]*

- Shelah's approach certainly looks a lot more like a kind of soundness failure than Hamkins.
- But let's suppose that we want to accept that this is another species of non-polar but sound question.
- But then we are pushed straight onto the falsity of the **Ignorance Norm**, since we very often know that a sentence is independent before we prove it to be.

- In some sense mathematics is very often like this.
- Usually, you know the answer before you've proved it.
- Even when you've got a proof of ϕ, continuing to inquire into whether ϕ may be desirable to shore up epistemic standpoint.
- There's a reason (beyond pragmatic mathematical concerns) why we accept new proofs of old theorems.
- **Natural response:** This is no different from other cases of improving epistemic standing (that the friends of these norms already consider).
- **Common move:** You're inquiring into subtly different questions.
- e.g. What a proof of ϕ looks like, rather than ϕ itself.

- **Problem 1:** It seems strikingly hard to delineate what an agent is doing.
- e.g. I am inquiring into whether there is a proofs that ϕ, using technique X,..
- Compare this to the much cleaner answer "Inquiring into ϕ/whether ϕ".
- **Problem 2:** There are some difficult issues here to do with unexpected results.
- Consider [Malliaris and Shelah, 2016]'s proof that $\mathfrak{p} = \mathfrak{t}$ (in ZFC!)
- (Well, don't really consider it, that proof is very hard, but here's the interesting thing...)
- It was independence that was expected and not the ZFC proof!

- This relates to a general phenomenon: Inquiring into questions that are either (a) thought to be unsound or (b) have known answers, can be very epistemically productive.
- Note that in mathematics, we can distinguish between *inquiring into* CH (the subject matter) and *inquiring into whether* CH (the question).
- I suggest that even if you're inquiring into the subject matter, this can bleed into inquiring into the question.
- Were Malliaris and Shelah not inquiring into the question of whether $\mathfrak{p} = \mathfrak{t}$ when they were inquiring into the subject matter of $\mathfrak{p} = \mathfrak{t}$?

- I just don't think it's plausible that inquiry and the specific goals of inquirers can be neatly delineated in the ways the **Knowledge Account** requires.
- Inquiry is broken up into many smaller "units".
- But it seems odd (and unnecessary) to prohibit a case of an unexpected result from being seen as an overarching inquiry into whether ϕ.
- And it is the **Epistemic Improvement Account** (and associated rejection of the relevant norms) that can support this characterisation of the activity.

- A standard move that my opponents make:
- We're not saying that agents always behave completely inappropriately when violating the **Ignorance** or **Soundness** norms there is just a respect in which they do.
- I'm happy with this (rather externalist) sense of inquiry.
- But look: There's an important kind of inquiry where those norms can be violated.
- Perhaps in the end there are just many different kinds of inquiry, knowledge directed and understanding directed.

- In this talk I've argued for the incorrectness of various norms surrounding inquiry (at least for certain kinds).
- But surely we want some norms?
- Here's a suggestion for a norm to go with the **epistemic improvement account**.

The Epistemic Optimality Norm.

If A has an interrogative attitude towards Q, then A should not be in an epistemically optimal situation with respect to Q.

- I hope it's clear that this is practically unattainable in a large number of cases.
- That's exactly how it should be.
- Perhaps the only sensible norms on inquiry are context sensitive.
- e.g. Have a control group, make steps in proofs clear, et cetera, et cetera...

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