To What Extent Does Formalisation Fix Meaning?

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To What Extent Does Formalisation Fix Meaning?

- A lot of my work is in the foundations of mathematics.
- But I'm also really interested in a wide range of areas.
- At the moment I'm thinking a lot about the philosophy of science, epistemology, conceptual engineering, and how this relates to the philosophy of language.

Main Aim.

Present the idea that even for mathematics, there are limitations to the extent to which formalisation fixes meaning.

Not-so-secret second aim.

Give a good commercial for a book I'm working on.

To What Extent Does Formalisation Fix Meaning?

- It's fairly common to think that formalising something doesn't entail fixing its meaning.
- But maybe for mathematics this is different.
- Isn't that what mathematicians do? Fix the meaning of their concepts/language using formalisation?
- We'll look at the particular case of the term "set" and conceptions of set (accounts of what the sets are).

To What Extent Does Formalisation Fix Meaning?

- A story about the history of set theory.
- Once upon a time, there was a logician named Gottlob Frege.
- He came up with a nice theory of sets viewed as concept extensions (called the naive conception) and showed how you could do all sorts of nice mathematical things.
- Unfortunately, his system was built out of straw, along came the big bad Bertrand Russell, and huffed, and puffed, and blew the house down with Russell's Paradox.
- Later, we figured out how to formalise mathematics using axioms, and figured out the right fix to Russell's paradox.
- We built a nice brick house out of the iterative conception (the idea that sets are formed in stages) and ZFC (our favourite theory of sets).
- And once we'd done this, we had a theory that fixed every sentence about the sets as true or false, and lived happily ever after.

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$$\begin{array}{c}\text{Naive } (\perp) \\ \big\downarrow \\ \text{Iterative } (\checkmark)\end{array}$$

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- Unfortunately, this really is a fairy tale.
- Really, the iterative conception and ZFC took a while to emerge, and one can trace out conflicting directions in which the community was pulled.
- Russell's paradox can be diagnosed as arising out of the following two principles (cf. [Incurvati, 2020]):

Universality.

A concept C is universal iff there exists a set of all the things falling under C.

Indefinite extensibility.

A concept C is indefinitely extensible iff whenever we succeed in defining a set u of objects falling under C, there is an operation which, given u, produces an object falling under C but not belonging to u.

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> *...in an attempt to make the history of the subject read more like an inevitable convergence on the one true religion, some authors have tried to find evidence of the iterative conception quite far back in the history of the subject. [Potter, 2004, p. 36]*

- Plausibly the iterative conception took a while to emerge, and there were periods (even post formalisation) where there were different underlying conceptions at play.
- But now that the iterative conception and ZFC have been accepted, our formalisation does yield determinate meaning, right?

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$$
\begin{tikzcd}
 & & \text{Naive } (\perp) \arrow[dl] \arrow[dr] & & \\
 & \text{Combinatorial} \arrow[dl] \arrow[d] & & \text{Logical} \arrow[dl] \arrow[d] \arrow[dr] & \\
\text{Graph} & \text{Iterative } (\checkmark) & \text{Stratified} & \text{Predicative} & \text{Iterative property}
\end{tikzcd}
$$

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- My work suggests that this isn't right either.
- Instead, I think that the iterative conception splits further down into multiple incompatible conceptions.
- In particular there's desires for (a) richness in the functions available, and (b) richness in closure properties that are in tension with one another (see [Barton, Ms] *Engineering Set-Theoretic Concepts* for details).

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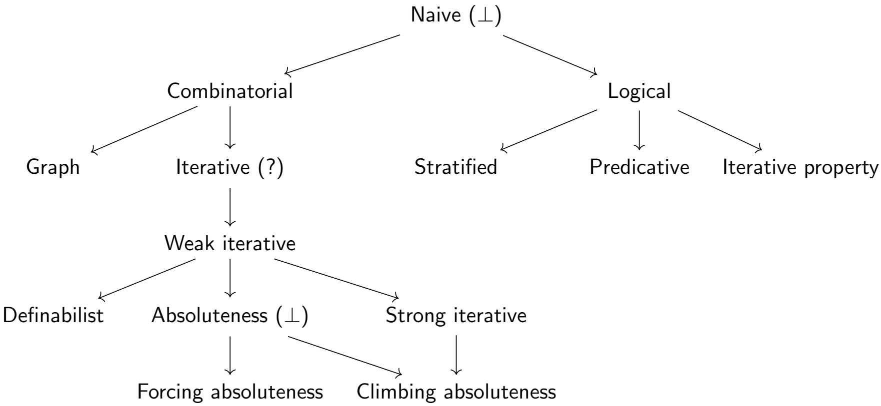

To What Extent Does Formalisation Fix Meaning?

- Here's the take home message:
- Formalisation doesn't straightforwardly fix meaning, formalisation plus a conception does.
- This is so even in mathematics, where things may be more conceptually messy than one might think.
- Whilst formalisation still has an important role to play, there's a failure of various internalist principles here (lots of people think they are working with a determinate conception).

Question.

How does this phenomenon interlink with other areas? (e.g. philosophy of language, epistemology, philosophy of science, conceptual engineering, ...)

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Thanks for listening!

References

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